

Exclusive evolution kernels in two-loop order: parity even sector.

A.V. Belitsky¹, D. Müller

*Institut für Theoretische Physik, Universität Regensburg
D-93040 Regensburg, Germany*

Abstract

We complete the construction of the non-forward evolution kernels in next-to-leading order responsible for the scale dependence of e.g. parity even singlet distribution amplitudes. Our formalism is designed to avoid any explicit two-loop calculations employing instead conformal and supersymmetric constraints as well as known splitting functions.

Keywords: evolution equation, two-loop exclusive kernels, supersymmetric and conformal constraints

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¹Alexander von Humboldt Fellow.

1 Introduction.

Hard exclusive processes [1], i.e. involving a large momentum transfer, provide a complimentary and an equally important information about the internal structure of hadrons to the one gained e.g. in deep inelastic scattering in terms of inclusive parton densities. By means of QCD factorization theorems [2] physical observables measurable in these reactions, i.e. form factors and cross sections, are expressed as convolution of a hard parton rescattering subprocess and non-perturbative distribution amplitudes [1] and/or skewed parton distributions [3, 4, 5, 6]. Moreover, it is implied that the main contribution to the latter comes from the lowest two-particle Fock state in the hadron wave function. The field-theoretical background for the study of the distribution amplitudes is provided by their expression in terms of matrix elements of non-local operators sandwiched between a hadron and vacuum states (or hadron states with different momenta in the case of skewed parton distributions)

$$\phi(x) = \frac{1}{2\pi} \int dz_- e^{ixz_-} \langle 0 | \varphi^\dagger(0) \varphi(z_-) | h \rangle. \quad (1)$$

Due to the light-like character of the path separating the partons, φ , the operator in Eq. (1) diverges in perturbation theory and thus requires renormalization which inevitably introduces a momentum scale into the game so that a distribution acquires a logarithmic dependence on it. This dependence is governed by the renormalization group which within the present context is cast into the form of Efremov-Radyushkin-Brodsky-Lepage (ER-BL) evolution equation [7, 8]

$$\frac{d}{d \ln Q^2} \phi(x, Q) = V(x, y | \alpha_s(Q)) \overset{\text{e}}{\otimes} \phi(y, Q), \quad \text{with} \quad \overset{\text{e}}{\otimes} = \int_0^1 dy. \quad (2)$$

Note that the restoration of the generalized skewed kinematics in perturbative evolution kernel, V , is unambiguous and straightforward [9]. Therefore, we discuss in what follows only the case when the skewedness of the process equals unity.

Recently, we have addressed the question of calculation of two-loop approximation for the exclusive evolution kernels and give our results for the parity-odd sector in Ref. [10]. The main tools of our analysis were the constraints coming from known pattern of conformal symmetry breaking in QCD and supersymmetric relations arisen from super-Yang-Mills theory. In the present note we address the flavour singlet parity even case which is responsible for the evolution of the vector distribution amplitude.

2 Anatomy of NLO evolution kernels.

Our derivation is based on the fairly well established structure of the ER-BL kernel in NLO. Up to two-loop order we have

$$V(x, y | \alpha_s) = \frac{\alpha_s}{2\pi} V^{(0)}(x, y) + \left(\frac{\alpha_s}{2\pi} \right)^2 V^{(1)}(x, y) + \mathcal{O}(\alpha_s^3), \quad (3)$$

with the purely diagonal LO kernel $\mathbf{V}^{(0)}$ in the basis of Gegenbauer polynomials and NLO one having the structure governed by the conformal constraints [11, 12, 13]

$$\mathbf{V}^{(1)} = -\dot{\mathbf{V}} \stackrel{\text{e}}{\otimes} \left(\mathbf{V}^{(0)} + \frac{\beta_0}{2} \mathbb{1} \right) - \mathbf{g} \stackrel{\text{e}}{\otimes} \mathbf{V}^{(0)} + \mathbf{V}^{(0)} \stackrel{\text{e}}{\otimes} \mathbf{g} + \mathbf{D} + \mathbf{G}. \quad (4)$$

Here the first three terms are induced by conformal symmetry breaking counterterms in the $\overline{\text{MS}}$ scheme. Contrary to the LO kernel $\mathbf{V}^{(0)}$, the so-called dotted kernel $\dot{\mathbf{V}}^{(0)}$ and the \mathbf{g} kernel are off-diagonal in the space of Gegenbauer moments. They have been obtained by a LO calculation [11, 12, 13]. The remaining two pieces are diagonal and are decomposed into the \mathbf{G}^V kernel which is related to the crossed ladder diagram and contains the most complicated structure in terms of Spence functions, while the \mathbf{D}^V kernel originates from the remaining graphs and can be represented as convolution of simple kernels.

One of the ingredients of the NLO result are the one-loop kernels. We use for them improved expressions of Ref. [12] which are completely diagonal in physical as well as unphysical spaces of moments. In the matrix form we have

$$\mathbf{V}^{(0)V}(x, y) = \begin{pmatrix} C_F [{}^{QQ}_V(x, y)]_+ & -2T_F N_f {}^{QG}_V(x, y) \\ C_F {}^{GQ}_V(x, y) & C_A {}^{GG}_V(x, y) - \frac{\beta_0}{2} \delta(x - y) \end{pmatrix}, \quad (5)$$

where $\beta_0 = \frac{4}{3}T_F N_f - \frac{11}{3}C_A$ and $C_A = 3$, $C_F = 4/3$, $T_F = 1/2$ for QCD. The structure of the kernels reflects the supersymmetry in $\mathcal{N} = 1$ super-Yang-Mills theory [14, 15, 16]

$$\begin{aligned} {}^{QQ}_V &\equiv {}^{QQ}_V{}^a + {}^{QQ}_V{}^b, & {}^{QG}_V &\equiv {}^{QG}_V{}^a + 2{}^{QG}_V{}^c, \\ {}^{GQ}_V &\equiv {}^{GQ}_V{}^a + 2{}^{GQ}_V{}^c, & {}^{GG}_V &\equiv [2{}^{GG}_V{}^a + {}^{GG}_V{}^b]_+ + 2{}^{GG}_V{}^c, \end{aligned} \quad (6)$$

where the functions v^i are defined in the following way

$${}^{AB}v^i(x, y) = \theta(y - x) {}^{AB}f^i(x, y) \pm \begin{cases} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{cases} \quad \text{for} \quad \begin{cases} A = B \\ A \neq B \end{cases}, \quad (7)$$

with (here and everywhere $\bar{x} \equiv 1 - x$)

$$\begin{aligned} \begin{Bmatrix} {}^{AB}f^a \\ {}^{AB}f^b \end{Bmatrix} &= \frac{x^{\nu(A)-1/2}}{y^{\nu(B)-1/2}} \begin{Bmatrix} 1 \\ \frac{1}{y-x} \end{Bmatrix}, \\ {}^{AA}f^c &= \frac{x^{\nu(A)-1/2}}{y^{\nu(A)-1/2}} \begin{Bmatrix} 2\bar{x}y \left[\frac{4}{3} - \ln(\bar{x}y) \right] + y - x \\ 2\bar{x}y + y - x \end{Bmatrix} \quad \text{for} \quad A = \begin{Bmatrix} Q \\ G \end{Bmatrix}, \\ {}^{AB}f^c &= \frac{x^{\nu(A)-1/2}}{y^{\nu(B)-1/2}} \begin{Bmatrix} 2x\bar{y} - \bar{x} \\ 2\bar{x}y - \bar{y} \end{Bmatrix} \quad \text{for} \quad A = \begin{Bmatrix} Q \\ G \end{Bmatrix} \neq B. \end{aligned} \quad (8)$$

The index $\nu(A)$ coincides with the index of the Gegenbauer polynomials in the corresponding channel, i.e. $\nu(Q) = 3/2$ and $\nu(G) = 5/2$. The eigenvalues of the same v^i -kernel in different

channels are related to each other (here $v_{jj} \equiv v_j$)

$$\begin{aligned} {}^{QQ}v_j^a &= -\frac{1}{6} {}^{QG}v_j^a = \frac{6}{j(j+3)} {}^{GQ}v_j^a = \frac{1}{2} {}^{GG}v_j^a = \frac{1}{(j+1)(j+2)}, \\ {}^{QQ}v_j^b &= {}^{GG}v_j^b - 1 = -2\psi(j+2) + 2\psi(1) + 2, \\ {}^{QQ}v_j^c &= -\frac{1}{6} {}^{QG}v_j^c = \frac{6}{j(j+3)} {}^{GQ}v_j^c = \frac{1}{3} {}^{GG}v_j^c = \frac{2}{j(j+1)(j+2)(j+3)}. \end{aligned} \quad (9)$$

Note that we have the identity ${}^{GQ}v_j^c = {}^{QQ}v_j^a/3 = {}^{GG}v_j^a/6$, which in the next section will serve as a guideline for the construction of $\dot{\mathbf{V}}$ and \mathbf{G} kernels.

3 Construction of $\dot{\mathbf{V}}$ and \mathbf{G} kernels.

To proceed further let us consider first the construction of the so-called dotted kernels whose off-diagonal conformal moments are simply expressed in terms of the one-loop anomalous dimensions, ${}^{AB}\gamma_j^{(0)}$, of the conformal operators as $\theta_{j-2,k}({}^{AB}\gamma_j^{(0)} - {}^{AB}\gamma_k^{(0)})d_{jk}$ with $d_{jk} = -\frac{1}{2}[1 + (-1)^{j-k}]\frac{(2k+3)}{(j-k)(j+k+3)}$. We introduce the matrix

$$\dot{\mathbf{V}}^{(0)V}(x, y) = \begin{pmatrix} C_F [{}^{QQ}\dot{v}(x, y)]_+ & -2T_F N_f {}^{QG}\dot{v}^V(x, y) \\ C_F {}^{GQ}\dot{v}^V(x, y) & C_A {}^{GG}\dot{v}^V(x, y) \end{pmatrix}, \quad (10)$$

where we use the decomposition analogous to Eqs. (5,6) for the LO kernels including the same “+”-prescription although this time the kernels are regular at the point $x = y$. The general structure of ${}^{AB}\dot{v}^i$ reads

$${}^{AB}\dot{v}^i(x, y) = \theta(y-x) {}^{AB}f^i(x, y) \ln \frac{x}{y} + \Delta {}^{AB}f^i(x, y) \pm \begin{cases} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{cases}, \quad \text{for} \quad \begin{cases} A = B \\ A \neq B \end{cases}. \quad (11)$$

For the dotted a and b -kernels we have $\Delta {}^{AB}f^i(x, y) \equiv 0$ with $i = a, b$. To find the dotted c -kernels we make use of the fact that kernels with the same conformal moments in different channels are related by differential equations owing to the following simple relations for the Gegenbauer polynomials

$$\frac{d}{dx} C_j^{3/2}(2x-1) = 6C_{j-1}^{5/2}(2x-1), \quad \frac{d}{dx} \frac{w(x|5/2)}{N_j(5/2)} C_{j-1}^{5/2}(2x-1) = -6 \frac{w(x|3/2)}{N_j(3/2)} C_j^{3/2}(2x-1), \quad (12)$$

where $w(x|\nu) = (x\bar{x})^{\nu-1/2}$ is the weight function and $N_j(\nu) = 2^{-4\nu+1} \frac{\Gamma^2(\frac{1}{2})\Gamma(2\nu+j)}{\Gamma^2(\nu)(\nu+j)j!}$ is the normalization coefficient. From the knowledge of conformal moments, which are determined by the eigenvalues of the corresponding kernels given in Eq. (9), and using the expansion of the kernels w.r.t. the Gegenbauer polynomials

$${}^{AB}v^i(x, y) = \sum_{j=0}^{\infty} \frac{w(x|\nu(A))}{N_j(\nu(A))} C_{j+3/2-\nu(A)}^{\nu(A)} (2x-1) {}^{AB}v_j^i C_{j+3/2-\nu(B)}^{\nu(B)} (2y-1)$$

we find then the following differential equations

$$\frac{d}{dy} {}^{GQ}\dot{v}^c(x, y) = {}^{GG}\dot{v}^a(x, y) + {}^{GG}\dot{v}^a(x, y), \quad (13)$$

$$\frac{d}{dx} {}^{GQ}\dot{v}^c(x, y) = -2 {}^{QQ}\dot{v}^a(x, y) - {}^{QQ}\dot{v}^a(x, y), \quad (14)$$

$${}^{QG}\dot{v}^c(x, y) = \frac{1}{3} \frac{d}{dx} {}^{GG}\dot{v}^c(x, y). \quad (15)$$

One of the entries in Eq. (11), namely

$$\Delta^{GG}\dot{f}^c(x, y) = 2 \frac{x^2}{y^2} (y - x), \quad (16)$$

has been obtained in Ref. [13]. Thus defined ${}^{GG}\dot{v}^c$ kernel possesses the correct conformal moments in both un- and physical sectors. Eqs. (13,14) allow us (after fixing the integration constant) to find $\Delta^{GQ}\dot{f}^c$, while from Eq. (15) we conclude that $\Delta^{QG}\dot{f}^c$ is trivially induced by $\Delta^{GG}\dot{f}^c$. Therefore, we have finally

$$\Delta^{GQ}\dot{f}^c = x^2(2x - 3) \ln \frac{x}{y}, \quad \Delta^{QG}\dot{f}^c = -\frac{x}{3y^2} (4x - 5y + 2xy). \quad (17)$$

Next the \mathbf{g} function is given by [12, 13]

$$\mathbf{g}(x, y) = \theta(y - x) \begin{pmatrix} -C_F \left[\frac{\ln(1-\frac{x}{y})}{y-x} \right]_+ & 0 \\ C_F \frac{x}{y} & -C_A \left[\frac{\ln(1-\frac{x}{y})}{y-x} \right]_+ \end{pmatrix} \pm \begin{Bmatrix} x \rightarrow \bar{x} \\ y \rightarrow \bar{y} \end{Bmatrix}, \quad (18)$$

with $(-) +$ sign corresponding to (non-) diagonal elements.

The construction of the diagonal $\mathbf{G}(x, y)$ kernel related to the crossed ladder diagrams is straightforward up to a number of points which are not obvious and present the most non-trivial part of the machinery. Let us give here its construction in more detail as compared to Ref. [10]. From the result in the flavour non-singlet sector [17, 18, 19] and the general limiting procedure to the forward case [3, 9] we know that all entries in the matrix

$$\mathbf{G}^i(x, y) = -\frac{1}{2} \begin{pmatrix} 2C_F \left(C_F - \frac{C_A}{2} \right) [{}^{QQ}\dot{G}^i(x, y)]_+ & 2C_A T_F N_f {}^{QG}\dot{G}^i(x, y) \\ C_F C_A {}^{GQ}\dot{G}^i(x, y) & C_A^2 [{}^{GG}\dot{G}^i(x, y)]_+ \end{pmatrix}, \quad (19)$$

must have the following general structure

$${}^{AB}\dot{G}^i(x, y) = \theta(y - x) \left({}^{AB}\dot{H}^i + \Delta {}^{AB}\dot{H}^i \right) (x, y) + \theta(y - \bar{x}) \left({}^{AB}\dot{\overline{H}}^i + \Delta {}^{AB}\dot{\overline{H}}^i \right) (x, y), \quad (20)$$

with the following expressions for H and \overline{H}

$${}^{AB}\dot{H}^i(x, y) = 2 \left[\pm {}^{AB}\dot{\overline{f}}^i (\text{Li}_2(\bar{x}) + \ln y \ln \bar{x}) - {}^{AB}\dot{f}^A \text{Li}_2(\bar{y}) \right], \quad (21)$$

$${}^{AB}\dot{\overline{H}}^i(x, y) = 2 \left[\left({}^{AB}\dot{f}^i \mp {}^{AB}\dot{\overline{f}}^i \right) \left(\text{Li}_2 \left(1 - \frac{x}{y} \right) + \frac{1}{2} \ln^2 y \right) + {}^{AB}\dot{f}^i (\text{Li}_2(\bar{y}) - \text{Li}_2(x) - \ln y \ln x) \right], \quad (22)$$

where the upper (lower) sign corresponds to the $A = B$ ($A \neq B$) channels. For the QQ sector we have $\Delta^{QQ}H = \Delta^{QQ}\overline{H} = 0$. However, in general these addenda are nonzero and are needed to ensure the diagonality of the kernels. From the known two-loop splitting functions we have to require as well that in the forward limit these terms contribute only to rational functions and/or terms containing single logs of momentum fractions.

The reduction $P \rightarrow V^D$ procedure from the forward to non-forward kinematics [12] is hard to handle for the restoration of ΔH contributions, so we have to rely on different principles. We do this by exploring supersymmetry and conformal covariance of $\mathcal{N} = 1$ super-Yang-Mills theory [15, 16]. As a matter of fact being wrong for all order results these assumptions hold true within the present context since the ^{AB}G kernels arise from the crossed ladder diagrams which have no UV divergent subgraphs and, therefore, require no renormalization. Thus, these kernels can be constructed from six constraints on anomalous dimensions of conformal operators. In principle these relations can also be written for the kernels in the ER-BL representation so that taking the known two entries of the quark-quark channel one can deduce all other channels. Unfortunately, at first glance it seems that not all of these constraints have a simple solution in the ER-BL representation. For this reason we modify our construction in the following way. Because of both supersymmetry and conformal covariance, the mixed channels are related by the equation

$$^{GQ}G^i(x, y) = \frac{(\bar{x}x)^2}{\bar{y}y} {}^{QG}G^i(y, x). \quad (23)$$

Employing this relation we can get a further one, from the so-called Dokshitzer supersymmetry constraint,

$$\frac{d}{dy} {}^{QQ}G^i(x, y) + \frac{d}{dx} {}^{GG}G^i(x, y) = -3 {}^{QG}G^i(x, y), \quad (24)$$

which allows to obtain the GG entry provided we already know the kernel in the mixed channel.

Let us consider first the parity odd sector. At LO we have for moments $^{QG}v_j^A = 6 {}^{QQ}v_j^a$. Thus we can simply obtain the QG kernel differentiating the QQ one. Fortunately, it turns out that the $^{QG}G^A$ kernel can be obtained² in the same way

$$^{QG}G^A(x, y) = \frac{d}{dy} {}^{QQ}G^a(x, y), \quad (25)$$

where $^{QQ}G^a$ is given by Eqs. (20-22) with $\Delta^{QQ}H^a = \Delta^{QQ}\overline{H}^a = 0$. The GQ entry is simply deduced from Eq. (23), while the GG one comes from the solution of the differential equation (24). The integration constant as a function of y is almost fixed by the necessary condition of diagonality

$$^{GG}G^A(x, y) = \frac{(x\bar{x})^2}{(y\bar{y})^2} {}^{GG}G^A(y, x). \quad (26)$$

²The correctness of this and subsequent results is checked by forming the Gegenbauer moments and comparing them with known NLO forward anomalous dimensions [20].

The remaining degree of freedom can be easily fixed from the requirement that the moments ${}^{GG}G_{ji}^A$ are diagonal for $i = 0, 1$. To simplify the result, we remove a symmetric function (w.r.t. the simultaneous interchange $x \rightarrow \bar{x}$ and $y \rightarrow \bar{y}$) which enters in both $\Delta^{GG}H^A$ and $\Delta^{GG}\bar{H}^A$ kernels, however with different overall signs and, therefore, disappears from ${}^{GG}G^A$. We present our final results in a symmetric manner as (cf. [10])

$$\Delta^{QQ}H^A(x, y) = \Delta^{QQ}\bar{H}^A(x, y) = 0, \quad (27)$$

$$\Delta^{QG}H^A(x, y) = \Delta^{QG}\bar{H}^A(\bar{x}, y), \quad \Delta^{QG}\bar{H}^A(x, y) = \frac{x\bar{x}}{(y\bar{y})^2} \Delta^{QG}\bar{H}^A(y, x) \quad (28)$$

$$\Delta^{GQ}H^A(x, y) = -\Delta^{GQ}\bar{H}^A(\bar{x}, y) \quad \Delta^{GQ}\bar{H}^A(x, y) = -2\frac{x\bar{x}}{y} \ln x + 2\frac{x\bar{x}}{\bar{y}} \ln y, \quad (29)$$

$$\Delta^{GG}H^A(x, y) = -\Delta^{GG}\bar{H}^A(\bar{x}, y), \quad (30)$$

$$\Delta^{GG}\bar{H}^A(x, y) = \frac{1 - 2x\bar{x}}{4y^2} + \frac{1 - 2\bar{x}(1 + \bar{x})}{4y^2} - 2\frac{x(\bar{x} + y - 3\bar{x}y)}{\bar{y}y^2} \ln x - 2\frac{\bar{x}(x + \bar{y} - 3x\bar{y})}{y\bar{y}^2} \ln y.$$

Now instead of dealing with the whole parity even sector, we can consider only the difference between vector and axial-vector functions

$$H^V = H^A + H^\delta. \quad (31)$$

In LO we have the simple equation ${}^{GQ}v_j^c = {}^{QQ}v_j^a/3 = {}^{GG}v_j^a/6$, see Eq. (9), which allows us to write down a simple relation between the kernels in different channels. However, to preserve the generic form of the ${}^{GQ}G^c$ function in the forward limit [20] we have used the following modified differential equations³:

$$\begin{aligned} \frac{d}{dx} {}^{GQ}H^c &= -4 \left({}^{QQ}H^a + 9 {}^{QQ}f^c \otimes^e {}^{QQ}f^c \right), & \frac{d}{dy} {}^{GQ}H^c &= 2 \left({}^{GG}H^a + 2 {}^{GG}f^c \otimes^e {}^{GG}f^c \right), \\ \frac{d}{dx} {}^{GQ}\bar{H}^c &= -4 \left({}^{QQ}\bar{H}^a + 9 {}^{QQ}\tilde{f}^c \otimes^e {}^{QQ}f^c \right), & \frac{d}{dy} {}^{GQ}\bar{H}^c &= 2 \left({}^{GG}\bar{H}^a + 2 {}^{GG}\tilde{f}^c \otimes^e {}^{GG}f^c \right), \end{aligned} \quad (32)$$

where $\tilde{f}^c(x, y) \equiv f^c(\bar{x}, y)$. The kernels ${}^{GG}H^a$ and ${}^{GG}\bar{H}^a$ are the parts of the whole parity odd functions derived in the fashion already explained above. The two sets of differential equations can be solved up to two integration constants which can easily be fixed from the diagonality of their conformal moments. Finally, we simplify the solution by adding pure diagonal pieces containing a and c kernels and their convolution as well as by removing symmetric terms which die out in ${}^{GQ}G$.

The entry in the QG channel can be obtained from the supersymmetric relation (23). To construct the GG kernel we use then the constraint (24) with ${}^{QQ}G^c \equiv 0$. We determine the integration constant as a function of x in the same manner as described previously. Our findings

³Note that we introduce a shorthand notation for the convolution, namely, ${}^{QQ}f^i \otimes^e {}^{QQ}f^j$ is understood as convolution of the corresponding ER-BL kernels and then keeping only the part proportional to $\theta(y - x)$.

for the $\Delta^{AB}H^\delta$ and $\Delta^{AB}\overline{H}^\delta$ can be summarized in the formulae

$$\Delta^{QG}H^\delta(x, y) = -\frac{x\bar{x}}{(y\bar{y})^2}\Delta^{QG}H^\delta(\bar{y}, \bar{x}), \quad \Delta^{QG}\overline{H}^\delta(x, y) = \frac{x\bar{x}}{(y\bar{y})^2}\Delta^{QG}\overline{H}^\delta(y, x), \quad (33)$$

$$\Delta^{GQ}H^\delta(x, y) = \Delta^{GQ}\overline{H}^\delta(\bar{x}, y) + 20\frac{x(x-\bar{x})}{3y} - 4\frac{\bar{x}(3+2\bar{x})}{3y}\ln\bar{x} + 4\frac{x(3+2x)}{3\bar{y}}\ln y, \quad (34)$$

$$\begin{aligned} \Delta^{GQ}\overline{H}^\delta(x, y) &= -\frac{61}{18} + 2x\bar{x}\left(1 - (3-10\bar{x})\ln y + (3-10x)\ln x\right) \\ &+ \frac{\bar{x}(6-19\bar{x}+6\bar{x}^2)}{3y} - 2\frac{\bar{x}(y+x(\bar{x}-x))}{\bar{y}}\ln y + 2\frac{x(\bar{y}+\bar{x}(x-\bar{x}))}{y}\ln x, \\ \Delta^{GG}H^\delta(x, y) &= \Delta^{GG}\overline{H}^\delta(\bar{x}, y) - \frac{20-18x+55x\bar{x}}{6y^2} - \frac{20-23x\bar{x}}{6\bar{y}^2} - \frac{17+32x+28x^2}{6y\bar{y}} \\ &- 2\frac{\bar{x}}{y}\left(2\frac{\bar{x}-x}{\bar{y}} + \frac{2+3\bar{x}}{y}\right)\ln\bar{x} - 2\frac{x}{\bar{y}}\left(2\frac{x-\bar{x}}{y} + \frac{2+3x}{\bar{y}}\right)\ln y, \\ \Delta^{GG}\overline{H}^\delta(x, y) &= -(1-x-y)\left(\frac{20-22x+21x\bar{x}}{6y^2} + \frac{20-22\bar{x}+21x\bar{x}}{6\bar{y}^2} + \frac{39+38x\bar{x}}{6y\bar{y}}\right) \\ &+ 2\left(\frac{x^3}{3y^2} - \frac{x^2(21-20x)}{3y} - 2\frac{x\bar{x}^2}{\bar{y}}\right)\ln x + 2\left(\frac{\bar{x}^3}{3\bar{y}^2} - \frac{\bar{x}^2(21-20\bar{x})}{3\bar{y}} - 2\frac{\bar{x}x^2}{y}\right)\ln y. \end{aligned} \quad (35)$$

4 Restoration of remaining diagonal terms.

As in the twist-two axial and transversity sectors [10] it turns out that the remaining diagonal piece, \mathbf{D}^V , can be represented as the convolution of simple diagonal kernels. To find it we take first the forward limit

$$\mathbf{P}(z) = \text{LIM } \mathbf{V}(x, y) \equiv \lim_{\tau \rightarrow 0} \frac{1}{|\tau|} \begin{pmatrix} QQV & \frac{1}{\tau} QGV \\ \frac{1}{z} GQV & \frac{1}{z} GG V \end{pmatrix}^{\text{ext}} \begin{pmatrix} z \\ \tau \\ \tau \end{pmatrix},$$

and compare our result with the known DGLAP kernel \mathbf{P}^V [20]. In this way,

$$\mathbf{D}^V(z) = \mathbf{P}^V(z) - \text{LIM} \left\{ -\dot{\mathbf{V}} \otimes \left(\mathbf{V}^{(0)V} + \frac{\beta_0}{2} \mathbb{1} \right) - \mathbf{g} \otimes \mathbf{V}^{(0)V} + \mathbf{V}^{(0)V} \otimes \mathbf{g} + \mathbf{G}^V \right\}, \quad (36)$$

we extract the remaining part $\text{LIM} \mathbf{D}^V(x, y)$ and find then the desired convolutions in the forward representation. Note that we map the antiparticle contribution, i.e. $z < 0$, into the region $z > 0$ by taking into account the underlying symmetry of the singlet parton distributions. Our findings can be immediately mapped back into the ER-BL representation:

$$\begin{aligned} {}^{QQ}D^V &= C_F^2 [D_F]_+ - C_F \frac{\beta_0}{2} [D_\beta]_+ - C_F \left(C_F - \frac{C_A}{2} \right) \left[\frac{4}{3} {}^{QQ}Q_V + 2 {}^{QQ}Q_V^a \right]_+ \\ &+ 4 C_F T_F N_f \left\{ \frac{1}{3} {}^{QQ}Q_V^a \otimes {}^{QQ}Q_V^a - 6 {}^{QQ}Q_V^c \otimes {}^{QQ}Q_V^c - {}^{QQ}Q_V^a + \frac{7}{6} {}^{QQ}Q_V^c \right\}, \end{aligned} \quad (37)$$

$${}^{QG}D^V = -C_F T_F N_f \left\{ 2 \left[{}^{QQ}Q_V \right]_+ \otimes {}^{QG}Q_V^c + {}^{QQ}Q_V^a \otimes {}^{QG}Q_V^a + \frac{3}{2} {}^{QG}Q_V^a + 6 {}^{QG}Q_V^a \right\} \quad (38)$$

$$\begin{aligned}
& + 2 C_A T_F N_f \left\{ - \left[\frac{8}{3} [Q Q_v]_+ + 56^{Q Q_v^c} \right] \overset{e}{\otimes} Q Q_v^c + \frac{130}{3} Q Q_v^a \overset{e}{\otimes} Q Q_v^a \right. \\
& \quad \left. + \left[\frac{55}{9} - 2\zeta(2) \right] Q Q_v^a - \left[\frac{301}{18} + 4\zeta(2) \right] Q Q_v^c \right\}, \\
^{GQ}D^V & = C_F^2 \left\{ - [^{GG}v^A]_+ \overset{e}{\otimes} \left[\frac{1}{2} ^{GQ}v^a + 3^{GQ}v^c \right] - 5^{GG}v^a \overset{e}{\otimes} ^{GQ}v^a - 3^{GQ}v^a \right\} \\
& - C_F \beta_0 \left\{ [^{GG}v^A]_+ \overset{e}{\otimes} \left[\frac{1}{2} ^{GQ}v^a + ^{GQ}v^c \right] + \frac{3}{4} ^{GG}v^a \overset{e}{\otimes} ^{GQ}v^a + \frac{5}{3} ^{GQ}v^a \right\} \\
& + C_F C_A \left\{ - [^{GG}v^A]_+ \overset{e}{\otimes} \left[^{GQ}v^a - \frac{3}{2} ^{GQ}v^c \right] - \frac{25}{6} ^{GG}v^a \overset{e}{\otimes} ^{GQ}v^a + 9^{GG}v^c \overset{e}{\otimes} ^{GQ}v^c \right. \\
& \quad \left. - \left(\frac{43}{9} + 2\zeta(2) \right) ^{GQ}v^a + \left(\frac{8}{9} - 4\zeta(2) \right) ^{GQ}v^c \right\}, \\
^{GG}D^V & = C_A^2 \left\{ [^{GG}v^A]_+ \overset{e}{\otimes} \left[^{GG}v^a + \frac{11}{3} ^{GG}v^c \right] - 14^{GG}v^a \overset{e}{\otimes} ^{GG}v^a + 12^{GG}v^c \overset{e}{\otimes} ^{GG}v^c \right. \\
& + \frac{2}{3} [^{GG}v^A]_+ - \frac{131}{12} ^{GG}v^a + \frac{91}{18} ^{GG}v^c - 2\delta(x-y) \left. \right\} \\
& - C_A \frac{\beta_0}{2} \left\{ -\frac{1}{2} ^{GG}v^a \overset{e}{\otimes} ^{GG}v^a + \frac{5}{3} [^{GG}v^V]_+ + 3^{GG}v^a + \frac{13}{3} ^{GG}v^a + 2\delta(x-y) \right\} \\
& + C_F T_F N_f \left\{ ^{GG}v^a \overset{e}{\otimes} ^{GG}v^a + \frac{4}{3} ^{GG}v^c - \delta(x-y) \right\}.
\end{aligned} \tag{39}$$

$$\begin{aligned}
& + \frac{2}{3} [^{GG}v^A]_+ - \frac{131}{12} ^{GG}v^a + \frac{91}{18} ^{GG}v^c - 2\delta(x-y) \left. \right\} \\
& - C_A \frac{\beta_0}{2} \left\{ -\frac{1}{2} ^{GG}v^a \overset{e}{\otimes} ^{GG}v^a + \frac{5}{3} [^{GG}v^V]_+ + 3^{GG}v^a + \frac{13}{3} ^{GG}v^a + 2\delta(x-y) \right\} \\
& + C_F T_F N_f \left\{ ^{GG}v^a \overset{e}{\otimes} ^{GG}v^a + \frac{4}{3} ^{GG}v^c - \delta(x-y) \right\}.
\end{aligned} \tag{40}$$

where D_F , D_β functions are known from the flavour non-singlet case [10]. In comparison to the parity odd sector the convolution of c -kernels appears as a new entry. It is worth mentioning that our result for the evolution kernels in the parity even singlet sector possesses the correct conformal moments in both the physical and unphysical sectors. This is to be contrasted with an explicit momentum fraction space calculation at LO and quark bubble insertions in NLO kernels for the mixed channels [12] where the improved kernels do not appear.

5 Conclusions.

To recapitulate the results presented here, we have reconstructed the two-loop singlet evolution kernels responsible for the scale dependence of the vector meson distribution amplitudes. We have avoided cumbersome next-to-leading calculations by adhering to an extensive use of the conformal and supersymmetric constraints derived earlier which thus play a paramount rôle in the formalism. The correctness of the results given here is proved by evaluating the Gegenbauer moments of the kernels which coincide with the anomalous dimensions derived in Ref. [13]. Our findings allow to use now the direct numerical integration (see Ref. [21, 22] for a leading order analysis of non-forward parton distributions) of the generalized exclusive evolution equations which provides a competitive alternative to the previously developed methods of orthogonal polynomial reconstruction of skewed parton distribution pursued by us in Ref. [23].

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References

- [1] S.J. Brodsky, G.P. Lepage, *Exclusive processes in Quantum Chromodynamics*, In *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller, World Scientific, Singapore (1989).
- [2] J.C. Collins, D.E. Soper, G. Sterman, *Factorization of hard processes in QCD*, In *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller, World Scientific, Singapore (1989).
- [3] D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, Fortschr. Phys. 42 (1994) 101.
- [4] X. Ji, Phys. Rev. D 55 (1997) 7114; J. Phys. G 24 (1998) 1181;
- [5] A.V. Radyushkin, Phys. Rev. D 56 (1997) 5524.
- [6] J.C. Collins, L. Frankfurt, M. Strikman, Phys. Rev. D 56 (1997) 2982.
- [7] A.V. Efremov, A.V. Radyushkin, Theor. Math. Phys. 42 (1980) 97; Phys. Lett. B 94 (1980) 245.
- [8] S.J. Brodsky, G.P. Lepage, Phys. Lett. B 87 (1979) 359; Phys. Rev. D 22 (1980) 2157.
- [9] F.-M. Dittes, B. Geyer, D. Müller, D. Robaschik, J. Hořejši, Phys. Lett. B 209 (1988) 325;
- [10] A.V. Belitsky, D. Müller, A. Freund, *Reconstruction of non-forward evolution kernels*, hep-ph/9904477.
- [11] D. Müller, Phys. Rev. D 49 (1994) 2525.
- [12] A.V. Belitsky, D. Müller, Nucl. Phys. B 527 (1998) 207.
- [13] A.V. Belitsky, D. Müller, Nucl. Phys. B 537 (1999) 397.
- [14] A.P. Bukhvostov, G.V. Frolov, E.A. Kuraev, L.N. Lipatov, Nucl. Phys. B 258 (1985) 601.
- [15] A.V. Belitsky, D. Müller, A. Schäfer, Phys. Lett. B 450 (1999) 126.
- [16] A.V. Belitsky, D. Müller, $\mathcal{N} = 1$ *supersymmetric constraints for evolution kernels*, hep-ph/9905211.
- [17] M.H. Sarmadi, Phys. Lett. B 143 (1984) 471.
- [18] F.-M. Dittes, A.V. Radyushkin, Phys. Lett. B 134 (1984) 359.

- [19] S.V. Mikhailov, A.V. Radyushkin, Nucl. Phys. B 254 (1985) 89.
- [20] W. Furmanski, R. Petronzio, Phys. Lett. B 97 (1980) 437;
 E.G. Floratos, C. Kounnas, R. Lacaze, Nucl. Phys. B 192 (1981) 417;
 R.K. Ellis, W. Vogelsang, *The evolution of parton distributions beyond leading order: singlet case*, hep-ph/9602356.
- [21] L. Frankfurt, A. Freund, V. Guzey, M. Strikman, Phys. Lett. B 418 (1998) 345; Phys. Lett. B 429 (1998) 414 (E).
- [22] I.V. Musatov, A.V. Radyushkin, *Evolution and models for skewed parton distributions*, hep-ph/9905376.
- [23] A.V. Belitsky, B. Geyer, D. Müller, A. Schäfer, Phys. Lett. B 421 (1998) 312;
 A.V. Belitsky, D. Müller, L. Niedermeier, A. Schäfer, Phys. Lett. B 437 (1998) 160; Nucl. Phys. B 546 (1999) 279;
 A.V. Belitsky, D. Müller, *Scaling violations and off-forward parton distributions: leading order and beyond*, hep-ph/9905263.